Low-Energy $\Xi-N$ Interactions*†

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The fraction $R_{\Lambda\Lambda}=\Lambda/(\Xi^0+\Lambda)$ in the reactions $\Xi^-+p\rightarrow\Lambda+\Lambda$ and $\Xi^-+p\rightarrow\Xi^0+n$ is calculated as a function of the $\Xi^--\Xi^0$ mass difference and the coupling constant combinations allowed by the octet model, by solving the Schrödinger equation for hard-core potentials together with contributions from pion, kaon, and K^* exchange. If $f=F/(D+F)$ in the mixture of the F- and D-type couplings between the baryons and the pseudoscalar mesons, then, using F-type $K^*\Lambda N$ and $K^*\Lambda\Xi$ couplings, we find that $R_{\Lambda\Lambda}$ lies in the range 0.01 to 0.15 for $0\leqslant f\leqslant 0.45$, increasing rapidly toward unity as $f\rightarrow 0.5$. Calculations of the effect of the closed Ξ-N channel on the low-energy $\Lambda-\Lambda$ scattering show that this is a very small effect.

1. INTRODUCTION

RECENT experimental evidence indicates that the spin of the Ξ (cascade hyperon) is $\frac{1}{2}$, and that $M(\Xi^-)$ is greater than $M(\Xi^0)$.[1] This at once means that the capture reaction

$$\Xi^-+p\rightarrow\Xi^0+n \qquad (1)$$

is allowed energetically and will compete with a process which has been previously discussed,[2,3]

$$\Xi^-+p\rightarrow\Lambda+\Lambda. \qquad (2)$$

The latter reaction is of particular interest in connection with the problem of determining the relative $\Xi-N$ parity; if the process is sufficiently frequent, the Ξ parity may be determined from an analysis of the polarization correlations of the final Λ particles. A simplifying feature occurs here in that the final state consists of two identical fermions, so that the Pauli principle limits the number of available final states. If we assume that the Stark-mixing effect[4] causes transitions to the $\Lambda-\Lambda$ system to occur primarily from S orbits, we find that for $S=1$ the transition is strictly forbidden for even $\Xi-N$ parity.

In the present work, a calculation is made of the rates of reactions (1) and (2) proceeding along the lines of the corresponding calculations for Σ^- capture in hydrogen which have been done by de Swart and Iddings.[5]

In the $^1S_0$ state, the $\Xi-N$ interaction also appears relevant to the S-wave $\Lambda-\Lambda$ interaction. Recently, Danysz *et al.*[6] have determined the binding energy of a $\Lambda\Lambda$ hypernucleus whose most probable interpretation is $_{\Lambda\Lambda}Be^{10}$. From this, it has been possible to estimate the strength of the S-wave $\Lambda-\Lambda$ interaction, with the conclusion that it is attractive and strong, although not strong enough to bind the $\Lambda-\Lambda$ system.[7–10] The $\Xi-N$ channel has threshold 26.4 MeV above the $\Lambda-\Lambda$ threshold and is linked with the $\Lambda-\Lambda$ channel by forces due to K meson and K^* exchange. Because of the short range of these forces, it may be that the $\Xi-N$ channel does not have much effect on $\Lambda-\Lambda$ scattering; on the other hand, for sufficiently strong K and K^* couplings, the effect of the $\Xi-N$ channel might not be negligible. We shall therefore investigate the effect which the closed $\Xi-N$ channel has upon $\Lambda-\Lambda$ scattering, by calculating the zero-energy scattering lengths and the phase shifts for the $^1S_0$ state as function of energy, both with and without the coupling to the closed $\Xi-N$ channel.

First, in Sec. 2, we shall derive the potential matrix for the two-channel system, using meson field theory. Several of the coupling constants involved are unknown, and the values predicted by the octet model will be used as a basis for the calculations. Then, in Sec. 3, we shall solve the coupled Schrödinger equations and determine first the K matrix and then the T-matrix elements of interest. These results are used to discuss the Ξ^--p capture processes. The effect of the $\Xi-N$ channel on the $\Lambda-\Lambda$ scattering is examined in Sec. 4. Finally, in Sec. 5, the conclusions are critically examined.

2. DERIVATION OF THE BARYON-BARYON POTENTIALS

The details of the procedure used to construct the potentials have been discussed by de Swart and Idd-

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[1] H. Ticho, Proceedings of the Brookhaven Conference on Weak Interactions, Brookhaven National Laboratory, September 1963 (to be published). A weighted average of the values reported gives $M(\Xi^0)=1316.3\pm1.2$ MeV/c^2, $M(\Xi^-)=1321.2\pm0.3$ MeV/c^2.

[2] L. B. Okun, I. Ia. Pomeranchuk, and I. M. Shmuskevich, Zh. Eksperim. i Teor. Fiz. **34**, 1246 (1958) [English transl.: Soviet Phys.—JETP **7**, 862 (1958)].

[3] S. B. Treiman, Phys. Rev. **113**, 355 (1959).

[4] T. Day, J. Sucher and G. Snow, Phys. Rev. Letters **3**, 61 (1959).

[5] J. J. de Swart and C. K. Iddings, Phys. Rev. **130**, 319 (1963).

[6] M. Danysz, K. Garbowska, J. Pniewski, T. Pniewski, J. Zakrewski, J. Lemonne, P. Renard, J. Sacton, D. O'Sullivan, J. Shah, A. Thompson, P. Allen, S. Heeman, A. Montwill, J. Allen, M. Beniston, D. Davis, D. Garbutt, V. Bull, R. Kumar, and P. March, Phys. Rev. Letters **11**, 29 (1963).

[7] R. Dalitz, Phys. Letters **5**, 53 (1963).

[8] H. Nakamura, Phys. Letters **6**, 207 (1963).

[9] A. Deloff, Phys. Letters **6**, 83 (1963).

[10] R. Dalitz and G. Rajasekaran, Nucl. Phys. (to be published).

ings,[11] and we shall not repeat them here. We will adopt their notation throughout, except as otherwise noted. We have for the nonrelativistic interaction Hamiltonian density

$$\begin{aligned}H_{\rm int}=&(f_{NN}/\mu)(4\pi)^{1/2}(N^\dagger\sigma_i\boldsymbol{\tau}N)\cdot\nabla_i\boldsymbol{\pi}\\&+(f_{\Lambda\Sigma}/\mu)(4\pi)^{1/2}(\Lambda^\dagger\sigma_i\boldsymbol{\Sigma}\cdot\nabla_i\boldsymbol{\pi}+\text{h.c.})\\&+(f_{\Xi\Xi}/\mu)(4\pi)^{1/2}(\Xi^\dagger\sigma_i\boldsymbol{\tau}\Xi)\cdot\nabla_i\boldsymbol{\pi}\\&+(f_{N\Lambda}/\mu)(4\pi)^{1/2}(N^\dagger\sigma_i\Lambda\nabla_iK+\text{h.c.})\\&+(f_{\Xi\Lambda}/\mu)(4\pi)^{1/2}(\Xi^\dagger\sigma_i\Lambda\nabla_iK^G+\text{H.c.}).\quad(3)\end{aligned}$$

The numerical values of the masses used in the calculation of the K matrices for the various spin and isospin states are, in MeV/c^2,

$$\begin{aligned}&\text{pion mass}=\mu\;=138.1 && M_\Xi=1318.5\\&\text{kaon mass}=\mu_K=496 && M_N=\;938.9\\&K^*\text{ mass}=\mu_V=888 && M_\Lambda=1115.36.\end{aligned}$$

These represent averages taken over the components of the various isomultiplets. The convention used for the isospinors is

$$\Xi=\begin{pmatrix}\Xi^0\\ \Xi^-\end{pmatrix},\quad K^G=\begin{pmatrix}K^0\\ -K^-\end{pmatrix}.\qquad(4)$$

Rationalized coupling constants are used, so that, e.g., $f_{NN}=0.285$.

In studying the Ξ^- capture reaction rates in hydrogen, the differences in masses between the members of the Ξ and N isomultiplets must be taken into account. In the method we shall use, the $\Xi-N$ scattering lengths at zero energy are first calculated for the $T=0$ and $T=1$ isotopic spin states, with neglect of these mass differences. The method of calculating the corrections arising from these mass differences will be described in Sec. 3. Since $T=0$ for the $\Lambda-\Lambda$ system, there can be no potential linking the $\Xi-N$ and $\Lambda-\Lambda$ channels for $T=1$. For even $\Xi-N$ parity, there will be a transition potential for $T=0$ for the $S=0$ spin state, but not for $S=1$. For the states $T=0$ and $T=1$, the potentials are constructed with the aid of the above Hamiltonian according to the Brueckner-Watson prescription[12] as the static limit of the scattering matrix.

The transition potential in the $T=0$, $S=0$ state also has a contribution arising from K^* exchange. It was found convenient to compute this as the static limit of the covariant second-order Feynman graph, using the Lagrangian density

$$\begin{aligned}L_{\rm int}/(4\pi)^{1/2}=&g_{\Lambda N}(M_\mu{}^+\bar{p}i\gamma_\mu\Lambda+M_\mu{}^0\bar{n}i\gamma_\mu\Lambda\\&+M_\mu{}^-\bar{\Lambda}i\gamma_\mu p+\bar{M}_\mu{}^0\bar{\Lambda}i\gamma_\mu n)\\&+g_{\Lambda\Xi}(\bar{M}_\mu{}^0\bar{\Xi}^0i\gamma_\mu\Lambda-M_\mu{}^-\bar{\Xi}^-i\gamma_\mu\Lambda\\&+M_\mu{}^0\bar{\Lambda}i\gamma_\mu\Xi^0-M_\mu{}^+\bar{\Lambda}i\gamma_\mu\Xi^-),\quad(5)\end{aligned}$$

where the symbol M_μ has been used to denote the vector K^* meson field.

The expressions for the potentials arising from one-pion, two-pion, one-kaon, and one-K^* exchange are

$$V(\Xi N,\Xi N)=-3f_{NN}f_{\Xi\Xi}V^{(2)}+f_{NN}{}^2f_{\Xi\Xi}{}^2[-3\,{}^XV^{(4)}+9\,{}^{II}V^{(4)}],\quad(6)$$

$$V(\Lambda\Lambda,\Xi N)=\sqrt{2}[f_{\Lambda N}f_{\Lambda\Xi}V_K{}^{(2)}+g_{\Lambda N}g_{\Lambda\Xi}V_V{}^{(2)}],\quad(7)$$

$$V(\Lambda\Lambda,\Lambda\Lambda)=3f_{\Lambda\Sigma}{}^4[{}^XV^{(4)}+{}^{II}V^{(4)}],\quad(8)$$

for $T=0$, and

$$V(\Xi N,\Xi N)=f_{NN}f_{\Xi\Xi}V^{(2)}+f_{NN}{}^2f_{\Xi\Xi}{}^2[5\,{}^XV^{(4)}+{}^{II}V^{(4)}]\quad(9)$$

for $T=1$. $V^{(2)}$ and $V_K{}^{(2)}$ represent the contributions arising from the one-pion and one-kaon exchange graphs. They are given by

$$V^{(2)}=[V_\sigma{}^{(2)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T{}^{(2)}(x)S_{12}]\mu\qquad(10)$$

and

$$V_K{}^{(2)}=[V_\sigma{}^{(2)}(x')\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T{}^{(2)}(x')S_{12}]\mu_K'(\mu_K'/\mu)^2.\quad(11)$$

Here[13] $x'=\mu_K'r$. The tensor operator S_{12} is defined by

$$S_{12}=3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2.$$

The functional forms of $V_\sigma{}^{(2)}(x)$ and $V_T{}^{(2)}(x)$ are given in Ref. 11. The quantity μ_K' is given by

$$\mu_K'=\mu_K[1-(\gamma_0/\mu_K)^2]^{1/2}\qquad(12)$$

in which[14]

$$\gamma_0=[M_\Xi(M_\Xi-M_\Lambda)+M_N(M_\Lambda-M_N)]/\times(M_N+M_\Xi).\quad(13)$$

The values $\mu_K'=0.920\,\mu_K$ and $x'=3.30\,x$ were used in the calculations.

$V_V{}^{(2)}$ represents the contribution of the one-K^* exchange graph to the transition potential in the ${}^1S_0$ state. It is given by[15]

$$V_V{}^{(2)}=\mu_V'[e^{-x''}/x'']\qquad(14)$$

with

$$\mu_V'=\mu_V[1-(\gamma_0/\mu_V)^2]^{1/2}=0.975\,\mu_V\qquad(15)$$

and

$$x''=\mu_V'r=6.27x.\qquad(16)$$

Following de Swart and Iddings,[11] we also ignored the mass difference corrections in constructing the fourth-order potentials. The contribution of the fourth-order

[11] J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).

[12] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

[13] One uses μ_k' instead of μ_k here because the range and strength of the transition potential must be corrected for the channel mass difference. This point is discussed in Ref. 11.

[14] This is for the case of zero kinetic energy in the $\Lambda-\Lambda$ channel. The difference between this value and the one obtained by letting the kinetic energy in the $\Xi-N$ channel go to zero is negligible.

[15] In the ${}^1S_0$ state there will be an additional central contribution of unknown strength coming from the Pauli term in the interaction Lagrangian. This was not considered here.

crossed graphs is

$$^{X}V^{(4)}=[^{X}V_1^{(4)}(x)+^{X}V_\sigma^{(4)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+^{X}V_T^{(4)}(x)S_{12}]\mu. \quad (17)$$

The contribution of the fourth-order uncrossed (neglecting intermediate states of two baryons only) graphs is

$$^{II}V^{(4)}=[^{II}V_1^{(4)}(x)+^{II}V_\sigma^{(4)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+^{II}V_T^{(4)}(x)S_{12}]\mu. \quad (18)$$

The radial dependences of the terms in (17) and (18) are listed in Ref. 11.

3. THE $\Xi^- - p$ CAPTURE PROCESSES

Because the reactions $\Xi^-+p\rightarrow\Lambda+\Lambda$ and $\Xi^-+p\rightarrow\Xi^0+n$ are exothermic reactions, the products $k\sigma(\Lambda)$ and $k\sigma(\Xi^0)$ are finite as k (momentum of Ξ^- particle) tends to zero. There exist two ways to calculate these quantities. In the method followed by de Swart and Iddings,[5] the mass differences between members of the Ξ and N multiplets are taken into account in setting up the Schrödinger equation in the "particle basis." Here, this would require the solution of a three-channel problem, for the $\Xi^- - p$, $\Xi^0 - n$, and $\Lambda - \Lambda$ channels. Details of this method will be found in Ref. 5.

Here, we have used a simpler procedure. It employs the method of "equivalent boundary conditions" to take the mass differences into account. A discussion of this method has been given by Dalitz[16] and will not be repeated here.

The procedure followed was first to solve numerically the Schrödinger equation for $T=0$, $S=0$ at zero relative momentum in the $\Xi - N$ channel to obtain the A matrix, given generally (here $l=0$) by

$$A=-k^{-(l+\frac{1}{2})}Kk^{-(l+\frac{1}{2})}, \quad (19)$$

where K denotes the reaction matrix and the matrix k is diagonal, with k_{ii} equal to the relative momentum in channel i (the $\Lambda - \Lambda$ channel is labeled 1, the $\Xi - N$ channel is labeled 2).

The complex zero-energy scattering length for the $\Xi - N$ channel is then given by[17]

$$a_0=+A_{22}-iA_{12}q[1/(1+iA_{11}q)]A_{12}, \quad (20)$$

where

$$q=2\mu_1\Delta, \quad (21)$$

$$\mu_1=M_\Lambda/2, \quad (22)$$

$$\Delta=M_\Xi+M_N-2M_\Lambda, \quad (23)$$

and the subscript on the scattering length refers to the isospin state $T=0$. We will also need the scattering length for the $S=0$, $T=1$ state, and the scattering lengths for the $S=1$ states for $T=0$ and 1. Since at this stage we do not take into account violations of isospin conservation, no absorption into the $\Lambda - \Lambda$ channel can occur for $T=1$; hence, the $T=1$ scattering lengths must be real. As was explained in Sec. 1, no absorption into the $\Lambda - \Lambda$ channel can occur in the $T=0$, $S=1$ state; hence, this scattering length is also real. These three additional scattering lengths are simply the 11 elements of the corresponding A matrices, calculated from the definition given above. Here, we shall use the sign convention in which a hard sphere of radius d has a zero-energy scattering length $+d$.

Using ϕ_1 and ϕ_0 to denote the radial wave functions in the $T=1$ and $T=0$ states, the boundary conditions become (in the zero-range approximation)

$$-(1/a_1)\phi_1(0)=\phi_1'(0), \quad (24)$$

$$-(1/a_0)\phi_0(0)=\phi_0'(0). \quad (25)$$

We let φ_- and φ_0 denote the states of definite charge, which are related to ϕ_0 and ϕ_1 by

$$\varphi_-=(\phi_1+\phi_0)/\sqrt{2}, \quad \varphi_0=(\phi_1-\phi_0)/\sqrt{2}. \quad (26)$$

In the $\Xi^- - p$ scattering problem φ_- and φ_0 have the form

$$\varphi_-=\sin kr/k+Te^{ikr}, \quad (27)$$

$$\varphi_0=\qquad Ce^{ik_0r}. \quad (28)$$

Upon substituting in the boundary conditions (24) and (25), we obtain the following set of complex equations for the values of T and C at zero $\Xi^- - p$ relative momentum.

$$-(1/a_1)(T+C)=1+ik_0C, \quad (29)$$

$$-(1/a_0)(C-T)=-1+ik_0C, \quad (30)$$

where

$$k_0=[2\mu(M(\Xi^-)+M(p)-M(\Xi^0)-M(N))]. \quad (31)$$

The values used in the calculations were, in MeV/c^2,

$$M(\Xi^-)=1321 \quad M(p)=938.21,$$
$$M(\Xi^0)=1315,\ 1316,\ 1318,$$
$$M(N)=938.9, \quad M(\Lambda)=1115.36.$$

For a given spin state, the S-wave cross sections for the $\Xi^- - p$ capture reactions are given by

$$k\sigma(\Xi^0n)=4\pi k_0|C|^2, \quad (32)$$

$$k\sigma(\Lambda\Lambda)=4\pi[\mathrm{Im}T-k_0|C|^2], \quad (33)$$

as k, the relative momentum in the $\Xi^- - p$ channel, tends to zero. The values of T and C obtained by solving (29) and (30) were substituted into (32) and (33) to yield the desired cross sections. From these cross sections, the branching ratios for spin states S

$$R_{\Lambda\Lambda}(S)=k\sigma_S(\Lambda)/(k\sigma_S(\Xi^0)+k\sigma_S(\Lambda)), \quad (34)$$

[16] R. Dalitz, *Strange Particles and Strong Interactions* (Oxford University Press, New York, 1962), Chaps. VI and VII. See also *The Nuclear Interactions of the Hyperons* (Oxford University Press, New York, 1963), Chap. 6.

[17] R. Dalitz, Eq. (6.41) of the first reference cited in Ref. 16. Note that Dalitz uses a different ordering of the channels than is used here, so that channels 1 and 2 must be interchanged.

were calculated. The total fraction $R_{\Lambda\Lambda}$ of the $\Xi^- - p$ S-state atoms decaying by the reaction $\Xi^- + p \to \Lambda + \Lambda$ is given by

$$R_{\Lambda\Lambda} = \tfrac{1}{4}R_{\Lambda\Lambda}(0) + \tfrac{3}{4}R_{\Lambda\Lambda}(1)\,, \quad (35)$$

assuming that the statistical weights of the $\Xi^- - p$ atoms formed in singlet and triplet states are $\frac{1}{4}$ and $\frac{3}{4}$, respectively, and that there is no appreciable spin-state mixing.[18]

The calculated cross sections and branching ratios are listed in Tables I–IV as functions of various values of the uncertain Ξ^0 mass, as well as of the $K^*\Lambda N$ and $K^*\Lambda\Xi$ coupling constants and f, the relative weight of the f-type couplings. The values of f chosen were $f=0$, 0.25, 0.45 and 0.5; the results for these values are listed in Tables I, II, III, and IV, respectively. The coupling constants are given in terms of f and the known πNN pseudoscalar coupling constant $g=3.87$ by

$$f_{\Xi\Xi} = -(1-2f)(g/19.09)\,, \quad (36)$$

$$f_{\Lambda N} = \frac{-(1+2f)}{\sqrt{3}}\frac{g}{14.84}\,, \quad (37)$$

$$f_{\Xi\Lambda} = -\frac{(1-4f)}{\sqrt{3}}\frac{g}{17.62}\,, \quad (38)$$

$$f_{\Lambda\Sigma} = \frac{2(1-f)}{\sqrt{3}}\frac{g}{16.70}\,. \quad (39)$$

Relations (36)–(39) have been given (for pseudoscalar coupling constants) by, e.g., Martin and Wali.[19] According to the vector theory of strong interactions,[20] the vector meson coupling of vector form with the baryons is limited naturally to the F-type. In this case, the $K^*\Lambda N$ and $K^*\Lambda\Xi$ couplings are of opposite sign and of the order of magnitude of the ρNN coupling.[20,21] This

TABLE I. Cross sections and branching ratios for $f=0$. $f_{\Xi\Xi} = -0.203$, $f_{\Lambda\Sigma}=0.267$, $f_{\Lambda N}=-0.151$, $f_{\Lambda\Xi}=-0.127$. The quantities $k\sigma_s$ are given in units of the Yukawa $(\hbar/\mu c)$. Core radius $x_0=0.35\,\mu^{-1}$. $\Delta M = M(\Xi^-)-M(\Xi^0)$.

$g_{\Lambda N}=-g_{\Lambda\Xi}$	$k\sigma_0(\Xi^0)$	$k\sigma_0(\Lambda)$	$k\sigma_1(\Xi^0)$	$R_{\Lambda\Lambda}=\frac{1}{4}R_{\Lambda\Lambda}(0)$
		$\Delta M=6.2$ MeV/c^2		
0.5	0.860	0.035	1.867	0.010
1.4	0.873	0.102	1.867	0.026
2.0	0.899	0.234	1.867	0.052
		$\Delta M=5.2$ MeV/c^2		
0.5	0.764	0.035	1.676	0.011
1.4	0.776	0.102	1.676	0.029
2.0	0.800	0.233	1.676	0.056
		$\Delta M=3.2$ MeV/c^2		
0.5	0.519	0.035	1.163	0.016
1.4	0.527	0.101	1.163	0.040
2.0	0.546	0.232	1.163	0.075

TABLE II. Cross sections and branching ratios for $f=0.25$, $f_{\Xi\Xi}=-0.102$, $f_{\Lambda\Sigma}=0.200$, $f_{\Lambda N}=-0.226$, $f_{\Lambda\Xi}=0$. The quantities $k\sigma_s$ are given in units of the Yukawa $(\hbar/\mu c)$. Core radius $x_0=0.35\,\mu^{-1}$. $\Delta M=M(\Xi^-)-M(\Xi^0)$.

$g_{\Lambda N}=-g_{\Lambda\Xi}$	$k\sigma_0(\Xi^0)$	$k\sigma_0(\Lambda)$	$k\sigma_1(\Xi^0)$	$R_{\Lambda\Lambda}=\frac{1}{4}R_{\Lambda\Lambda}(0)$
		$\Delta M=6.2$ MeV/c^2		
0.5	0.319	0.0002	0.0525	0.0001
1.4	0.322	0.0102	0.0525	0.0076
2.0	0.332	0.0432	0.0525	0.0290
		$\Delta M=5.2$ MeV/c^2		
0.5	0.283	0.0002	0.0466	0.0002
1.4	0.286	0.0101	0.0466	0.0085
2.0	0.295	0.0432	0.0466	0.0319
		$\Delta M=3.2$ MeV/c^2		
0.5	0.193	0.0002	0.0317	0.0002
1.4	0.195	0.0100	0.0317	0.0122
2.0	0.201	0.0426	0.0317	0.0437

TABLE III. Cross sections and branching ratios for $f=0.45$, $f_{\Xi\Xi}=-0.0202$, $f_{\Lambda\Sigma}=0.147$, $f_{\Lambda N}=-0.286$, $f_{\Lambda\Xi}=0.101$. The quantities $k\sigma_s$ are given in units of the Yukawa $(\hbar/\mu c)$. Core radius $x_0=0.35\,\mu^{-1}$, $\Delta M=M(\Xi^-)-M(\Xi^0)$.

$g_{\Lambda N}=-g_{\Lambda\Xi}$	$k\sigma_0(\Xi^0)$	$k\sigma_0(\Lambda)$	$k\sigma_1(\Xi^0)$ $(\times 10^{-3})$	$R_{\Lambda\Lambda}=\frac{1}{4}R_{\Lambda\Lambda}(0)$
		$\Delta M=6.2$ MeV/c^2		
0.5	0.015	0.0192	0.94	0.142
1.4	0.014	0.0050	0.94	0.066
2.0	0.014	0.0001	0.94	0.002
		$\Delta M=5.2$ MeV/c^2		
0.5	0.013	0.0191	0.84	0.148
1.4	0.013	0.0050	0.84	0.072
2.0	0.013	0.0001	0.84	0.002
		$\Delta M=3.2$ MeV/c^2		
0.5	0.009	0.0191	0.57	0.170
1.4	0.009	0.0049	0.57	0.091
2.0	0.009	0.0001	0.57	0.003

TABLE IV. Cross sections and branching ratios for $f=0.5$, $f_{\Xi\Xi}=0.000$, $f_{\Lambda\Sigma}=0.134$, $f_{\Lambda N}=-0.301$, $f_{\Lambda\Xi}=0.127$. The quantities $k\sigma_s$ are given in units of the Yukawa $(\hbar/\mu c)$. Core radius $x_0=0.35\,\mu^{-1}$. $\Delta M=M(\Xi^-)-M(\Xi^0)$.

$g_{\Lambda N}=-g_{\Lambda\Xi}$	$k\sigma_0(\Xi^0)$ $(\times 10^{-4})$	$k\sigma_0(\Lambda)$	$k\sigma_1(\Xi^0)$	$R_{\Lambda\Lambda}=R_{\Lambda\Lambda}(0)$
		$\Delta M=6.2$ MeV/c^2		
0.5	0.5787	0.0307	0.000	0.9981
1.4	0.0708	0.0124	0.000	0.9994
2.0	0.0020	0.0013	0.000	0.9998
		$\Delta M=5.2$ MeV/c^2		
0.5	0.5172	0.0307	0.000	0.9983
1.4	0.0632	0.0124	0.000	0.9995
2.0	0.0018	0.0013	0.000	0.9999
		$\Delta M=3.2$ MeV/c^2		
0.5	0.3557	0.0307	0.000	0.9988
1.4	0.0435	0.0124	0.000	0.9996
2.0	0.0012	0.0013	0.000	0.9999

[18] For a detailed discussion of spin-state mixing, see D. E. Neville, Phys. Rev. **130**, 327 (1963).

[19] A. Martin and K. Wali, Phys. Rev. **130**, 2455 (1963).

[20] J. Sakurai, Proceedings of the International School of Physics "Enrico Fermi," Villa Monastero, Varenna, Como, Italy (to be published).

[21] M. Gell-Mann, California Institute of Technology, Synchrotron Laboratory Report, CSTL-20, 1961 (unpublished).

latter coupling constant has been estimated to be approximately $\sqrt{2}$;[20] the trial values $g_{\Lambda N}=0.5$, 1.4, and 2.0 were chosen, and in accordance with our assumption of F-type coupling, the corresponding values chosen for $g_{\Lambda\Xi}$ were equal but of opposite sign. This difference in sign between the $K^*\Lambda N$ and $K^*\Lambda\Xi$ couplings means that the vector-meson contribution to the transition potential in the $S=0$, $T=0$ state will reinforce the kaon contribution for $K\Lambda N$ couplings of the same sign as $K\Lambda\Xi(f=0)$ and tend to cancel the kaon contribution if the $K\Lambda N$ and $K\Lambda\Xi$ couplings are of opposite sign $(f=0.5)$.[22] In turn, this will be reflected in the behavior of $k\sigma_0(\Lambda)$ as a function of $g_{\Lambda\Xi}$. This behavior can be observed in the calculated results. Compare, e.g., the monotonic increase of $k\sigma_0(\Lambda)$ with increasing $g_{\Lambda N}$ in Table I $(f=0)$ with the monotonic decrease in $k\sigma_0(\Lambda)$ as $g_{\Lambda N}$ is increased in Table IV $(f=0.5)$.

Martin and Wali[23] have given an estimate for the value of f; it is $0.25\leqslant f\leqslant 0.45$. It is seen from the present results that these two limits on f correspond to widely different calculated values of $R_{\Lambda\Lambda}(0)$, the $\Lambda/(\Xi^0+\Lambda)$ ratio for $S=0$. For example, taking $g_{\Lambda N}=1.4$ and $M(\Xi^0)=1316$ gives $3.41\%\leqslant R_{\Lambda\Lambda}(0)\leqslant 28.6\%$. $R_{\Lambda\Lambda}$ itself does not attain as large a value within these limits on f; here, $0.85\%\leqslant R_{\Lambda\Lambda}\leqslant 7.2\%$. The large values of $R_{\Lambda\Lambda}$ as f approaches the value 0.5 (for $f=0.5$, as can be seen in Table IV, $R_{\Lambda\Lambda}$ becomes almost unity) follow from the fact that $f_{\Xi\Xi}=0$ for $f=0.5$; in this case the $\Xi-N$ interaction is just a hard core of radius $0.35\ \mu^{-1}$ and the only difference between the $\Xi-N$ scattering in the $T=0$ and $T=1$ states for $S=0$ arises from the small amount of absorption in the $\Lambda-\Lambda$ channel for $T=0$. As a result, the charge exchange will be very small,[24] and so the $\Xi^-+p\rightarrow\Lambda+\Lambda$ reaction can be expected to be the dominant process. For $f=0.5$, $a_1=a_0$ in the $S=1$ state, leading to zero charge exchange; because of isotopic spin conservation no $\Lambda-\Lambda$ production is possible either; hence, for $f=0.5$, $R_{\Lambda\Lambda}=R_{\Lambda\Lambda}(0)$. The charge-exchange reaction for $f=0.5$, $S=1$ is not completely forbidden in all approximations, however; it can be produced, e.g., by K exchange. For the case $f=0.5$, $S=0$, whether one can expect competition between the decay processes

$$\Xi^-+p\rightarrow\Xi^0+n\,, \tag{40}$$

and

$$\Xi^-+p\rightarrow\Lambda+\Lambda+\gamma\,, \tag{41}$$

is not clear without a detailed calculation.

The variation of $k\sigma(\Xi^0 n)$ with the three different values of the mass difference between Ξ^--p and Ξ^0-n systems is in good agreement with the phase-space estimate. This behavior can be readily understood in terms of the weakness of the pionic couplings of the cascade particle, resulting in very small scattering lengths. See, e.g., the expression for $k\sigma(\Xi^0 n)$ in Ref. 24.

TABLE V. $^1S_0$ $\Lambda\Lambda$ scattering length in F. Core radius given by $x_0=0.35\ \mu^{-1}$.

	$\Lambda\Lambda$ channel only	Coupled $\Lambda\Lambda$ and ΞN channels. $g_{\Lambda N}=0.5$ $g_{\Lambda\Xi}=-0.5$	$g_{\Lambda N}=1.4$ $g_{\Lambda\Xi}=-1.4$	$g_{\Lambda N}=2.0$ $g_{\Lambda\Xi}=-2.0$
$f=0$	-0.0150	-0.020	-0.033	-0.060
$f=0.25$	0.394	0.393	0.392	0.385
$f=0.5$	0.480	0.475	0.479	0.479

4. EFFECT OF THE CLOSED $\Xi-N$ CHANNEL ON THE LOW-ENERGY $\Lambda-\Lambda$ SCATTERING

In this section, a calculation is made of the low-energy $\Lambda-\Lambda$ scattering both with and without inclusion of effects arising from the $\Xi-N$ channel. de Swart[25] has calculated the $\Lambda-\Lambda$ scattering in the $^1S_0$ state, taking into account the coupling to the $\Sigma-\Sigma$ channel (the threshold for this channel lies 153.8 MeV above the $\Lambda-\Lambda$ threshold), with neglect of the coupling with the $\Xi-N$ channel. It is not obvious how large the effect of this last coupling may be, since its magnitude is influenced by two factors, (i) the short range of the K and K^* exchange forces coupling the two channels, and (ii) the K^* and K couplings to the baryons may be much stronger than has usually been considered likely. To investigate this, we shall neglect the $\Lambda-\Lambda$ coupling to the $\Sigma-\Sigma$ channel; it is well known that this coupling is not expected to be very significant in the $^1S_0$ state, since the one-pion exchange contribution is weak in singlet states, while the two-pion exchange contribution depends on $f_{\Sigma\Sigma}$,[4] which is expected to be small.[26]

The results of the calculation are presented in Tables V and VI. Table V gives the scattering lengths

TABLE VI. $^1S_0$ $\Lambda\Lambda$ Scattering lengths, with neglect of coupling to the $\Sigma\Sigma$ and ΞN channels.

x_0, μ^{-1}	$f=0$, $f_{\Lambda\Sigma}=0.267$ $a_{\Lambda\Lambda}$, F	$f=0.25$, $f_{\Lambda\Sigma}=0.200$ $a_{\Lambda\Lambda}$, F
0.35	-0.0150	0.394
0.34	-0.120	0.370
0.33	-0.260	0.344
0.32	-0.461	0.317
0.31	-0.782	0.288
0.30	-1.406	0.255
0.29	-3.26	0.219
0.28	135.5	0.179
0.27	3.71	0.132
0.26	1.98	0.075

[22] There is an extra minus sign in the kaon contribution arising from the fact that $\sigma_1\cdot\sigma_2=-3$ in the singlet state.

[23] A. Martin and K. Wali, Bull. Am. Phys. Soc. **8**, 515 (1963).

[24] This can be made clear by solving Eqs. (29) and (30) for C and substituting in (32), to give

$$k\sigma(\Xi^0 n)=\pi k_0|a_1-a_0|^2/|D|^2,$$

where

$$D=1+(i/2)(a_1+a_0)k_0.$$

[25] J. J. de Swart, Phys. Letters **5**, 58 (1963).

[26] Taking the $\Sigma-\Sigma$ channel into account, de Swart (Ref. 25) calculated the value $a_0=-5.2$ F, using $x_0=0.35\ \mu^{-1}$, $f_{\Lambda\Sigma}=0.312$, $f_{\Sigma\Sigma}=0$; for these same values, and with neglect of the $\Sigma-\Sigma$ channel, we obtained the value $a_0=-4.55$ F.

calculated with and without inclusion of the $\Xi-N$ channel, for $f=0$, 0.25, and 0.5. A hard-core radius of $0.35\ \mu^{-1}$ was used in all of the potentials, as before. The values of $f_{\Lambda\Sigma}$ are 0.267, 0.200, and 0.134 for $f=0$, 0.25, and 0.5, respectively.

We immediately see from Table V that the above choices of x_0 and $f_{\Lambda\Sigma}$ do not fit the experimental value for the $\Lambda-\Lambda$ scattering length, which is -1.76 ± 0.33 F.[27]

As can be seen in Sec. 2, $V_{\Lambda\Lambda}$ has a repulsive core with an attractive tail. For the isolated $\Lambda-\Lambda$ channel, therefore, one would expect the zero-energy scattering length to correspond to a net repulsive force for small $f_{\Lambda\Sigma}$ and to a net attractive force for $f_{\Lambda\Sigma}$ sufficiently large. Table V shows that this is indeed the case. For $f=0.5$, where $f_{\Lambda\Sigma}$ is only 0.134, a_0 for the isolated channel is 0.48 F. (For a pure hard core of radius $0.35\ \mu^{-1}$, it would be 0.50 F.) For $f=0$, the larger value of $f_{\Lambda\Sigma}$ is sufficient to make the net force weakly attractive, with $a_0=-0.015$ F.

For all three choices of f, the effect of the $\Xi-N$ channel is quite small. The effect is largest for $f=0$, as is to be expected since the K^*- and $K-$ exchange contributions to the off-diagonal elements of the potential matrix are both present and are of the same sign. (For $f=0.25$, only K^* exchange is present; for $f=0.5$ both are present, but have opposite sign.) The effect of the closed channel is always such as to increase the net $\Lambda-\Lambda$ attraction. This enhanced attraction is only sufficient to decrease the net repulsion for $f=0.25$ and $f=0.5$; even for $f=0$, it is well below that necessary for the existence of a bound $\Lambda-\Lambda$ state. For $f=0$, a bound $^1S_0$ state would be formed, provided the hard-core radius is smaller than $0.29\ \mu^{-1}$; however, this is still not sufficient for a bound $\Lambda-\Lambda$ state if $f=0.25$ holds. This is shown by the results given in Table VI, in which values of a_0 (calculated without inclusion of effects due to the $\Xi-N$ channel) are listed for $f=0$ ($f_{\Lambda\Sigma}=0.267$) and $f=0.25$ ($f_{\Lambda\Sigma}=0.200$) as a function of x_0, the hard-core radius.

The $\Lambda-\Lambda$ $^1S_0$ phase shifts were calculated as a function of incident Λ energy for energies up to the $\Xi-N$ threshold, and the effect of the $\Xi-N$ channel was found to be negligible ($<1.5°$) for all cases examined.

It is seen, therefore, from these calculations (based on the coupling constants of unitary symmetry) that the closed $\Xi-N$ channel has a negligible effect on the low-energy $\Lambda-\Lambda$ scattering, for values of f in the range 0 to 0.5.

[27] Deduced by Dalitz and Rajasekaran (Ref. 10) from data of the $_{\Lambda\Lambda}Be^{10}$ event observed by Danysz *et al.* (Ref. 6).

5. SUMMARY

The basic conclusion to be drawn from the results of Sec. 3 is that the $\Lambda/(\Xi^0+\Lambda)$ fraction $R_{\Lambda\Lambda}$ for $\Xi^- - p$ reactions in hydrogen is expected to be appreciable. This estimate is based on: (a) the qualitative correctness of the relations between the baryon-baryon-pseudoscalar meson coupling parameters predicted by unitary symmetry, and (b) certain reasonable assumptions as to the size of the hard cores in the baryon-baryon forces and the strength of the K^*-baryon couplings. The value of $R_{\Lambda\Lambda}$ is found to depend on the value of f, the $F/(D+F)$ proportion in the mixture of F- and D-type couplings. Our results predict a value for $R_{\Lambda\Lambda}$ to be roughly of the order 0.01–0.15 for $0\leqslant f\leqslant 0.45$, increasing rapidly toward unity as $f\rightarrow 0.5$.

As for the influence of the closed $\Xi-N$ channel on the $\Lambda-\Lambda$ scattering, this is seen to be quite small, according to the results of Sec. 4. The neglect of the $\Xi-N$ channel in such calculations as de Swart's[25] is, therefore, well justified.

The size of the repulsive core represents a major uncertainty in the above results, which are to be regarded as illustrative only. If the same core radius is used for all the potentials, as would be consistent with the hypothesis of a neutral vector meson coupled universally with all baryons, then it seems unlikely that small changes in this core radius would affect the qualitative conclusions of this calculation very much. On the other hand, the qualitative results could be modified very significantly by the choice of different core radii for different components of the potentials.

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